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Quantum Statistics from Oscillatory Sampling

A Detection-Theoretic Derivation of the Born Rule

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Abstract

We derive the Born rule and quantum interference from an oscillatory field model using standard signal processing and detection theory. Physical particles are modeled as coherent patterns in an underlying oscillatory field $\Phi(x,t) = \mathrm{Re}[\Psi(x,t)e^{-i\omega_c t}]$, where Ψ is a slowly varying envelope. Measurement is formalized as finite-window demodulation followed by threshold detection in the presence of noise. We show that the detection probability $P_{\text{click}} \propto |\Psi|^2$ emerges as the leading-order term in a Taylor expansion, with explicit higher-order corrections $O(|\Psi|^4)$ providing falsifiable predictions. Quantum interference arises automatically from superposition of same-frequency components. We demonstrate that the Heisenberg uncertainty relations are equivalent to the Gabor limit from signal processing—uncertainty is aliasing, not metaphysical mystery. The Schrödinger equation emerges as the non-relativistic envelope dynamics of an oscillatory field satisfying the Klein-Gordon equation. This paper addresses single-system detection statistics only; multi-particle entanglement and Bell inequality violations require additional theoretical structure not claimed here.

Keywords: Born rule, quantum measurement, oscillatory sampling, detection theory, demodulation, threshold detection, Heisenberg uncertainty, Nyquist-Shannon

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Scope and Claims

What this paper claims:

- The Born rule $P \propto |\psi|^2$ is derivable from oscillatory sampling + threshold detection
- Quantum interference emerges automatically from phase correlations
- Heisenberg uncertainty IS the Gabor/Nyquist limit from signal processing
- Higher-order corrections $O(|\psi|^4)$ provide falsifiable predictions

What this paper does NOT claim:

- Solution to Bell inequality violations or entanglement
- Explanation of spin or identical particle statistics
- Commitment to what the oscillatory field "really is" ontologically

Contents

1 Introduction

The Born rule—that measurement probabilities equal the squared modulus of the wave function amplitude (Born, 1926)—is typically introduced as an axiom of quantum mechanics. Despite nearly a century of interpretational work, there remains no consensus on *why* probabilities should be quadratic in amplitude rather than linear, cubic, or any other function.

This paper derives the Born rule from first principles using three ingredients:

1. An underlying oscillatory field structure
2. Finite-resolution demodulation (matched filtering)
3. Threshold detection in the presence of noise

Each ingredient is standard physics or signal processing. The novelty lies in connecting them into a coherent derivation that explains why $|\psi|^2$ specifically appears.

1.1 The Core Claim

We propose that physical particles are not point-like objects but coherent oscillatory patterns in an underlying field. What we call the "wave function" ψ is the slowly varying envelope of a high-frequency carrier oscillation. Measurement involves sampling this oscillatory structure at finite resolution, and detection occurs via threshold crossing in a noisy environment.

Under these assumptions, we show:

$$P_{\text{click}}(x) = P_{\text{dark}} + C|\Psi(x)|^2 + O(|\Psi(x)|^4) \tag{1}$$

The Born rule is the leading non-trivial term. The higher-order corrections are in principle observable at high field intensities.

1.2 Relation to Prior Work

The Born rule has been the subject of numerous derivation attempts across distinct research programs. Situating our approach within this landscape is essential to clarify what is genuinely novel.

1.2.1 Axiomatic and Information-Theoretic Derivations

The oldest rigorous derivation is *Gleason's theorem* (Gleason, 1957), which shows that in Hilbert spaces of dimension ≥ 3, the only probability measure consistent with the lattice structure of projections is the Born rule. While mathematically powerful, Gleason's result assumes the Hilbert space framework as given and does not explain *why* probability should depend on squared amplitudes physically.

Busch's extension (Busch, 2003) generalises Gleason's result to positive operator-valued measures (POVMs) and, crucially, to dimension 2, where Gleason's original proof fails. Saunders (Saunders, 2004) derives the Born rule from operational assumptions that apply even when probabilities are defined for a single resolution of the identity, weakening the structural demands of Gleason's approach.

More recently, Zurek's *envariance* program (Zurek, 2005) derives Born's rule from the symmetries of entangled quantum states within the no-collapse (Everettian) framework. The Deutsch-Wallace decision-theoretic approach (Deutsch, 1999; Wallace, 2003) argues that a rational agent in a branching universe must assign probabilities following the Born rule. Both approaches are internal to the quantum formalism: they derive Born's rule *given* quantum mechanics, rather than from sub-quantum structure.

A distinct programme seeks to reconstruct the entire quantum formalism from operational or informational axioms. Hardy (Hardy, 2001) showed that quantum theory is the unique alternative to classical probability theory satisfying five reasonable axioms including continuity of state transformations.

Chiribella, D'Ariano, and Perinotti (Chiribella et al., 2011) derived quantum theory from purely informational principles—causality, purification, and local distinguishability—demonstrating that the Born rule follows necessarily from the structure of information processing in a purifiable theory. These results are powerful: they show that *if* one accepts certain informational constraints, the Born rule is the only consistent probability assignment. Our approach differs in that we derive the Born rule from a physical model of detection rather than from abstract informational constraints, and in doing so we obtain specific predictions (the $O(|\Psi|^4)$ corrections) that the axiomatic approaches exclude by construction.

The Born rule has also attracted renewed attention in recent work. Hossenfelder (Hossenfelder, 2020) derives the Born rule from the requirement that probability be independent of the number of degrees of freedom. Lim (Lim, 2023) provides a simple derivation from unitary symmetry alone. Neumaier (Neumaier, 2025) offers a centennial review that traces the Born rule from its 1926 origin to modern POVM formulations, including a derivation from the definition of a quantum detector that shares some structural features with our detection-theoretic approach. From the opposing philosophical direction, QBism (Fuchs et al., 2014) treats the Born rule not as a physical law to be derived but as a normative constraint on rational belief updating—quantum probabilities are subjective, and the Born rule is a consistency requirement rather than an empirical discovery. Our derivation of Born's rule as a physical consequence of detection statistics stands in direct contrast to this position.

Our approach is fundamentally different from all of the above. We derive the Born rule from prequantum ingredients—oscillatory fields, signal processing, and threshold detection—without presupposing the Hilbert space structure. The quadratic dependence emerges from the physics of energy-based detection rather than from symmetry postulates, informational axioms, or normative constraints.

1.2.2 Stochastic Electrodynamics

Stochastic electrodynamics (SED) attempts to recover quantum behavior from classical electrodynamics supplemented with a real, physical zero-point radiation field. Initiated by Marshall and Boyer (Boyer, 1975), and developed extensively by de la Peña and Cetto (de la Peña and Cetto, 1996; de la Peña et al., 2015), SED shows that vacuum fluctuations can account for atomic stability and certain quantum statistics.

Our oscillatory field model shares SED's commitment to an underlying continuous field ontology and its program of deriving quantum statistics from classical-field-plus-noise structures. However, we differ in a critical respect: SED retains point-particle dynamics driven by stochastic fields, whereas our model treats particles as *patterns* in the field itself. Additionally, our derivation mechanism is detection-theoretic rather than dynamical—the Born rule emerges from measurement statistics, not from stochastic equations of motion.

1.2.3 Threshold Detection and Classical Random Fields

Our work is most closely related to Khrennikov's prequantum classical statistical field theory (PCSFT) (Khrennikov, 2009a, 2012b). Khrennikov models quantum particles as classical random fields and derives Born's rule by formalizing detection as threshold crossing of field intensity. His 2009 paper explicitly demonstrates that $P \propto |\Psi|^2$ emerges as the leading term in weak-signal threshold detection with additive noise—a result we arrive at independently through a different route. Notably, Khrennikov has extended PCSFT beyond single-system statistics, showing that threshold detection of classical random signals can reproduce violations of CHSH-type inequalities (Khrennikov, 2012a)—a result directly relevant to the Bell limitation we discuss in Section 11.3.

La Cour and Williamson (La Cour and Williamson, 2020) provide a parallel development in quantum optics, showing that the Born rule emerges from amplitude threshold detection applied to classical-like optical fields with vacuum fluctuations. Their model produces detection statistics matching quantum predictions for single-photon and two-photon experiments.

Our approach differs from both in three respects. First, we ground the derivation in the explicit *demodulation* step (matched filtering), making the connection to signal processing concrete rather than abstract. Second, we derive the Schrödinger equation from Klein-Gordon envelope dynamics, providing a dynamical foundation that Khrennikov's and La Cour's models lack. Third, our sinc^2 resolution-dependence analysis (Section 6) and the explicit connection of Heisenberg uncertainty to the Gabor limit (Section 7) are, to our knowledge, novel contributions.

1.2.4 De Broglie and Pilot-Wave Theory

De Broglie's original matter-wave hypothesis (de Broglie, 1924) proposed that particles are associated with physical oscillations at the Compton frequency $\omega_0 = mc^2/\hbar$. Our model takes this intuition seriously: particles are not objects that *have* waves but ARE the wave patterns. This is closer to de Broglie's original vision than to the Copenhagen interpretation or even to the later de Broglie-Bohm pilot-wave theory (Bohm, 1952), which retains point particles guided by a wave. We dispense with the guiding equation entirely; the particle IS the coherent envelope.

Within pilot-wave theory, Valentini's quantum equilibrium programme (Valentini, 1991) demonstrates that the Born rule can emerge dynamically: a sub-quantum H-theorem drives arbitrary initial distributions toward $|\psi|^2$ equilibrium. This is structurally parallel to our derivation—both approaches treat the Born rule as an emergent statistical regularity rather than a fundamental postulate—though the mechanisms differ (relaxation dynamics versus detection-theoretic filtering). Recent numerical work by Hardel, Hervieux, and Manfredi (Hardel et al., 2023) confirms Born rule emergence in Nelson's stochastic dynamics, showing that interference patterns appear only after quantum equilibrium is established, which is consistent with our picture of interference as a consequence of detection statistics applied to coherent superpositions.

A striking physical analogue is provided by the walking droplet experiments of Couder and Fort (Couder and Fort, 2006), in which macroscopic oil droplets bouncing on a vibrating fluid bath exhibit single-particle diffraction and interference patterns. These are classical systems in which an oscillatory substrate generates quantum-like statistics through the interaction of a localised excitation with its own wave field—precisely the type of mechanism our theoretical model formalises at the quantum scale.

1.2.5 Photodetection Theory

The quantum theory of photodetection, developed by Glauber (Glauber, 1963) and Mandel (Mandel, 1977; Mandel and Wolf, 1995), establishes that detection probabilities depend on normally ordered field correlation functions—effectively on $|E^{(+)}|^2$ for single detections. The foundational work of Helstrom (Helstrom, 1967) and Kelley and Kleiner (Kelley and Kleiner, 1964) formalised detection theory within quantum mechanics. Helstrom's subsequent monograph (Helstrom, 1976) remains the standard treatment of quantum detection and estimation theory, providing the mathematical framework within which our detection model operates—though we invert its logical direction.

Our framework inverts this relationship. Rather than *assuming* quantum mechanics and deriving detection statistics, we start from detection physics and derive quantum statistics. The intensity-based detection model is shared, but the logical direction is reversed.

1.2.6 Decoherence and the Quantum-Classical Transition

Our resolution-dependent coherence analysis (Section 6) connects to the decoherence programme (Zurek, 2003; Schlosshauer, 2007; Joos et al., 2003). Standard decoherence theory explains the emergence of classical behaviour through entanglement with environmental degrees of freedom, which suppresses off-diagonal density matrix elements. Our Section 6.3 arrives at a compatible result through a different mechanism: frequency differences between superposed components cause interference terms to average away under temporal sampling, yielding effective decoherence without invoking environmental entanglement.

This mechanism is closely related to the consistent (or decoherent) histories framework initiated by Griffiths (Griffiths, 1984) and developed by Gell-Mann and Hartle (Gell-Mann and Hartle, 1993). In that framework, probabilities are assigned to coarse-grained histories only when decoherence conditions are satisfied—that is, when interference between alternative histories is negligible. Our resolution-dependent analysis arrives at a structurally similar conclusion from a different direction: coarse temporal sampling naturally suppresses cross-frequency interference terms, so that well-defined detection probabilities emerge precisely when the decoherence condition is met. The connection suggests that the consistent histories framework may find a natural physical grounding in the detection-theoretic picture we develop here.

1.2.7 Not Digital Physics

It is worth distinguishing our approach from digital physics or cellular automaton interpretations ('t Hooft, 2016). We do not claim that the universe computes or is fundamentally discrete. The oscillatory field Φ is a continuous, real-valued function. Any discrete notation is shorthand for oscillation phase. That said, our approach shares with 't Hooft's programme the goal of deriving quantum statistics from a deterministic sub-quantum layer; the disagreement concerns the proposed mechanism (continuous oscillatory fields and detection-theoretic filtering versus cellular automata and information loss), not the ambition.

1.3 Outline

Section 10 is new to this version and presents four concrete numerical simulation designs that can validate the paper's central claims without requiring laboratory experiments. Section 2 defines the oscillatory field model. Section 3 formalizes measurement as demodulation. Section 4 introduces threshold detection and derives the Born rule. Section 5 demonstrates interference emergence. Section 6 analyzes resolution dependence. Section 7 connects Heisenberg uncertainty to the Nyquist-Gabor limit. Section 8 derives the Schrödinger equation from Klein-Gordon dynamics. Section 9 presents falsifiable predictions. Section 10 presents the simulation designs. Section 11 discusses limitations and future work.

2 The Oscillatory Field Model

2.1 Field Definition

Definition 2.1 (Oscillatory Field)**.** The physical field at spacetime point (x,t) is:

$$\Phi(x,t) = \mathrm{Re}\left[\Psi(x,t)e^{-i\omega_c t}\right] \tag{2}$$

where ω_c is a high carrier frequency ($\omega_c \gg$ observable energy scales) and $\Psi(x,t)$ is the complex-valued, slowly varying envelope.

The envelope Ψ varies slowly compared to the carrier period $2\pi/\omega_c$. This separation of scales is the standard slowly varying envelope approximation (SVEA) from nonlinear optics and plasma physics.

2.2 Particles as Patterns

Definition 2.2 (Particle)**.** A "particle" is a localized, coherent modulation of the oscillatory field—a region where the envelope Ψ has significant amplitude and maintains phase coherence.

This differs fundamentally from point-particle ontology. Particles are not objects that *have* waves; particles ARE the wave patterns. This is closer to de Broglie's original matter-wave intuition than to Copenhagen's particle-wave duality.

2.3 What This Is Not

This is **not** digital physics or cellular automaton interpretation ('t Hooft, 2016). We are not claiming the universe computes or is made of information bits. The field Φ is a continuous, real-valued function. Any discrete notation (such as ± 1 for phase opposition) is shorthand for oscillation phase, not fundamental digital ontology.

2.4 Superposition

For multiple sources or paths, the envelope superposes:

$$\Psi(x,t) = \sum_j \psi_j(x,t) \tag{3}$$

Each ψ_j represents a contribution from path j. The total field is:

$$\Phi = \mathrm{Re}\left[\left(\sum_j \psi_j\right) e^{-i\omega_c t}\right] \tag{4}$$

This superposition principle is inherited from linearity of the underlying wave equation.

3 Measurement as Demodulation

3.1 Why Naive Averaging Fails

One might attempt to define measurement as time-averaging the field:

$$\langle \Phi \rangle_{\Delta t} = \frac{1}{\Delta t} \int_{t_0}^{t_0+\Delta t} \Phi(x,t')\, dt' \tag{5}$$

For $\Delta t \gg 2\pi/\omega_c$ (many oscillation cycles), this integral averages to approximately zero because Φ oscillates symmetrically about zero. Naive averaging destroys all information.

3.2 Demodulation via Matched Filtering

The correct approach is *demodulation*: extracting the envelope by mixing with the carrier frequency and filtering.

Definition 3.1 (Sampled Complex Amplitude)**.** The sampled complex amplitude at position x, centered at time t_0, with window function g of duration Δt, is:

$$A(x;t_0) = \int_{-\infty}^{\infty} g(t-t_0)\, \Phi(x,t)\, e^{+i\omega_c t}\, dt \tag{6}$$

The factor $e^{+i\omega_c t}$ shifts the carrier to baseband. The window g (normalized: $\int g\, dt = 1$) provides temporal localization. Common choices include rectangular (boxcar) or Gaussian windows.

3.3 Evaluation of the Integral

Substituting Eq. (2) into Eq. (6):

$$\begin{aligned} A &= \int g(t-t_0) \cdot \frac{1}{2}\left(\Psi e^{-i\omega_c t} + \Psi^* e^{+i\omega_c t}\right) \cdot e^{+i\omega_c t}\, dt \\ &= \frac{1}{2}\int g(t-t_0)\left(\Psi + \Psi^* e^{+2i\omega_c t}\right) dt \end{aligned} \tag{7}$$

In the SVEA regime where Ψ varies slowly and $\Delta t \gg 2\pi/\omega_c$, the $e^{+2i\omega_c t}$ term averages to zero:

$$A(x;t_0) \approx \frac{1}{2}\Psi(x,t_0) \tag{8}$$

The sampled amplitude is proportional to the envelope at the measurement time.

3.4 Noise

Real measurements include noise from thermal fluctuations, detector electronics, and vacuum fluctuations. We model this as additive complex Gaussian noise:

$$A = \mu + \eta, \quad \eta \sim \mathscr{CN}(0,\sigma^2) \tag{9}$$

where $\mu = \frac{1}{2}\Psi(x,t_0)$ is the signal and η is circular complex Gaussian noise with variance σ^2.

This noise model is standard in communication theory and photodetection (Rice, 1944; Mandel and Wolf, 1995).

4 Threshold Detection and the Born Rule

4.1 Detection as Threshold Crossing

Physical detectors do not measure continuous field amplitudes; they produce discrete "clicks" or counts. We model this as threshold crossing:

Definition 4.1 (Detection Event)**.** A detection event ("click") occurs when the measured intensity exceeds a threshold:

$$\text{click} \iff |A|^2 > \Theta \tag{10}$$

This model applies to photomultipliers, Geiger counters, CCDs, and essentially any counting detector (Helstrom, 1967; Kelley and Kleiner, 1964; Mandel, 1977). The threshold Θ depends on detector characteristics.

4.2 Statistics of the Measured Intensity

With $A = \mu + \eta$ where $\eta \sim \mathscr{CN}(0,\sigma^2)$, the quantity $|A|^2$ follows a noncentral chi-squared distribution with 2 degrees of freedom (equivalently, a Rice or Rician intensity distribution).

The probability of detection is:

$$P_{\text{click}}(x) = \Pr\left(|A|^2 > \Theta\right) = \Pr\left(|\mu+\eta|^2 > \Theta\right) \tag{11}$$

This is a smooth function of $|\mu|^2$.

4.3 Taylor Expansion: Deriving the Born Rule

Theorem 4.1 (Born Rule from Threshold Detection)**.** In the weak-signal regime ($|\mu|^2 \ll \sigma^2$ or $|\mu|^2 \ll \Theta$), the detection probability expands as:

$$P_{\text{click}} = P_{\text{dark}} + C(\Theta, \sigma)\,|\mu|^2 + O(|\mu|^4) \tag{12}$$

where P_{dark} is the dark count rate (noise-only detections) and $C > 0$ is a coefficient depending on threshold and noise level.

Proof. Let $\eta = \eta_R + i\eta_I$ where $\eta_R, \eta_I \sim \mathcal{N}(0, \sigma^2/2)$ independently. Then:

$$|A|^2 = |\mu + \eta|^2 = |\mu|^2 + 2\text{Re}(\mu^*\eta) + |\eta|^2 \tag{13}$$

For small $|\mu|$, expand P_{click} around $\mu = 0$. At $\mu = 0$:

$$P_{\text{dark}} = \Pr(|\eta|^2 > \Theta) \tag{14}$$

The first derivative with respect to $|\mu|^2$ involves $\text{Re}(\mu^*\eta)$, which has zero mean. The leading correction comes from the second-order term.

More directly: model the detection probability as depending on the signal-to-noise ratio. For threshold detection with Gaussian noise, the tail probability of a noncentral chi-squared variable has the expansion:

$$P_{\text{click}} = P_{\text{dark}} \left(1 + \frac{|\mu|^2}{\sigma^2} f(\Theta/\sigma^2) + O(|\mu|^4/\sigma^4)\right) \tag{15}$$

where $f > 0$ for reasonable threshold settings.

Absorbing constants:

$$P_{\text{click}} = P_{\text{dark}} + C|\mu|^2 + O(|\mu|^4) \tag{16}$$

■

4.4 The Born Rule

Since $\mu = \frac{1}{2}\Psi$:

$$\boxed{P_{\text{click}}(x) = P_{\text{dark}} + C'|\Psi(x)|^2 + O(|\Psi(x)|^4)} \tag{17}$$

This is the Born rule. The detection probability is proportional to $|\Psi|^2$ at leading order, with higher-order corrections that become relevant at high intensities.

The result is independent of the specific threshold Θ or noise level σ^2—these only affect the proportionality constant C'.

4.5 Why Quadratic?

The quadratic dependence $|\Psi|^2$ rather than $|\Psi|$ or $|\Psi|^4$ arises because:

1. Detection depends on *intensity* $|A|^2$, not amplitude $|A|$
2. The linear term $\text{Re}(\mu^*\eta)$ averages to zero
3. The leading non-vanishing term is quadratic in μ

This is not arbitrary—it follows from the physics of energy-based detection in noisy environments.

5 Interference

5.1 Two-Path Superposition

Consider two coherent contributions to the envelope:

$$\Psi(x) = \psi_1(x) + \psi_2(x) \tag{18}$$

The intensity is:

$$\begin{aligned} |\Psi|^2 &= |\psi_1 + \psi_2|^2 \\ &= |\psi_1|^2 + |\psi_2|^2 + 2\mathrm{Re}(\psi_1^* \psi_2) \end{aligned} \tag{19}$$

The cross-term $2\mathrm{Re}(\psi_1^* \psi_2)$ is the interference term.

5.2 Double-Slit Pattern

For equal-amplitude paths with a position-dependent phase difference $\delta(x)$:

$$\psi_1 = Ae^{i\phi_1}, \quad \psi_2 = Ae^{i\phi_2}, \quad \delta = \phi_2 - \phi_1 \tag{20}$$

The total envelope:

$$\Psi = A(e^{i\phi_1} + e^{i\phi_2}) = 2A\cos(\delta/2)\, e^{i(\phi_1+\phi_2)/2} \tag{21}$$

The intensity:

$$|\Psi|^2 = 4A^2\cos^2(\delta/2) = 2A^2(1+\cos\delta) \tag{22}$$

This produces the characteristic fringe pattern:

$$P_{\text{click}}(x) - P_{\text{dark}} \propto 1 + \cos\delta(x) \tag{23}$$

Maxima occur where $\delta = 2\pi n$ (constructive interference); minima where $\delta = (2n+1)\pi$ (destructive interference).

5.3 Which-Path Information

If a measurement determines which path the particle took, the superposition $\psi_1 + \psi_2$ is replaced by a statistical mixture: ψ_1 with probability p_1, or ψ_2 with probability p_2.

The expected intensity becomes:

$$\langle|\Psi|^2\rangle = p_1|\psi_1|^2 + p_2|\psi_2|^2 \tag{24}$$

No cross-term appears—interference is destroyed. This is complementarity, derived from the detection model rather than postulated.

5.4 N-Path Generalization

For N coherent paths:

$$|\Psi|^2 = \left|\sum_{j=1}^{N} \psi_j\right|^2 = \sum_j |\psi_j|^2 + \sum_{j\neq k} \psi_j^* \psi_k \tag{25}$$

All pairwise interference terms appear, recovering the full quantum interference formalism.

6 Resolution Dependence

6.1 The Sinc Factor

For a rectangular window of duration Δt, the demodulation integral gives:

$$A = \frac{1}{2}\Psi \cdot e^{i\omega_c(t_0+\Delta t/2)} \cdot \mathrm{sinc}\left(\frac{\omega_c \Delta t}{2}\right) \tag{26}$$

where $\mathrm{sinc}(x) = \sin(x)/x$.

The intensity scales as:

$$|A|^2 \propto |\Psi|^2 \cdot \mathrm{sinc}^2\left(\frac{\omega_c \Delta t}{2}\right) \tag{27}$$

6.2 Resolution Regimes

Define the characteristic resolution scale $\Delta t^* = \pi/\omega_c$.

- **High resolution** ($\Delta t \ll \Delta t^*$): The sinc factor ≈ 1. Individual oscillation phases are resolved. Behavior appears classical-like.
- **Low resolution** ($\Delta t \gg \Delta t^*$): The sinc factor oscillates and averages effects. Aliasing occurs. Quantum superposition statistics emerge.
- **Transition** ($\Delta t \sim \Delta t^*$): Partial resolution. Coherence is partially preserved.

The sinc^2 factor quantifies the transition continuously—there is no sharp quantum/classical boundary.

6.3 Multi-Frequency Superposition

For a superposition of different frequencies (energy eigenstates):

$$\Phi = \sum_n c_n \phi_n(x) e^{-i\omega_n t} \tag{28}$$

After demodulation at a reference frequency ω_c, cross-terms between different frequencies $\omega_n \neq \omega_m$ acquire rapidly oscillating phase factors $e^{i(\omega_n-\omega_m)t_0}$.

Averaging over measurement times t_0:

$$\langle e^{i(\omega_n-\omega_m)t_0} \rangle_{t_0} = \delta_{nm} \tag{29}$$

The cross-terms vanish, leaving:

$$\langle |\Psi|^2 \rangle_{t_0} = \sum_n |c_n|^2 |\phi_n|^2 \tag{30}$$

This explains decoherence (Zurek, 2003; Schlosshauer, 2007): interaction with an environment introduces frequency differences (entanglement with environmental degrees of freedom), causing interference terms to average away. The result is structurally equivalent to the decoherence condition in the consistent histories framework (Griffiths, 1984; Gell-Mann and Hartle, 1993): probabilities are well-defined precisely when cross-terms between distinct histories (here, distinct frequency components) vanish under coarse-graining.

7 Uncertainty as Aliasing

7.1 Energy-Time Uncertainty

The Heisenberg energy-time uncertainty relation states:

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2} \tag{31}$$

Using $E = \hbar\omega$:

$$\hbar\Delta\omega \cdot \Delta t \geq \frac{\hbar}{2} \implies \Delta\omega \cdot \Delta t \geq \frac{1}{2} \tag{32}$$

This is the Gabor limit from signal processing. A signal of duration Δt has frequency uncertainty at least $\Delta\omega \geq 1/(2\Delta t)$. This is a mathematical theorem about Fourier transforms, not quantum magic.

7.2 Position-Momentum Uncertainty

The position-momentum uncertainty relation:

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2} \tag{33}$$

Using $p = \hbar k$ (de Broglie relation):

$$\Delta x \cdot \hbar\Delta k \geq \frac{\hbar}{2} \implies \Delta x \cdot \Delta k \geq \frac{1}{2} \tag{34}$$

This is spatial frequency uncertainty. A wave packet localized to region Δx has wavenumber spread at least $\Delta k \geq 1/(2\Delta x)$. Again, Fourier mathematics.

7.3 The Nyquist Connection

The Nyquist-Shannon sampling theorem (Nyquist, 1928; Shannon, 1949) states that to resolve a frequency ω, you need sampling rate $r > 2\omega$, equivalently measurement duration:

$$\Delta t < \frac{\pi}{\omega} \tag{35}$$

A measurement of duration Δt cannot distinguish frequencies within a band of width $\sim 1/\Delta t$. States whose energies differ by less than $\hbar/\Delta t$ are "aliased" into apparent superposition.

> **Key Insight:** Heisenberg uncertainty IS Nyquist/Gabor uncertainty (Gabor, 1946; Shannon, 1949). Quantum uncertainty is not metaphysical weirdness—it is the inevitable consequence of finite-resolution sampling of oscillatory structure.

8 Schrödinger Equation from Klein-Gordon

8.1 The Klein-Gordon Equation

If the underlying oscillatory field satisfies the Klein-Gordon equation (the relativistic wave equation for a scalar field):

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2}\right)\Phi = 0 \tag{36}$$

8.2 Envelope Extraction

For a non-relativistic particle, the dominant time dependence is rest-mass oscillation at frequency $\omega_0 = mc^2/\hbar$. Factorize:

$$\Phi(x,t) = \psi(x,t)\, e^{-imc^2t/\hbar} \tag{37}$$

where ψ varies slowly compared to $e^{-imc^2t/\hbar}$.

8.3 Derivation

Computing derivatives:

$$\frac{\partial\Phi}{\partial t} = \left(\frac{\partial\psi}{\partial t} - \frac{imc^2}{\hbar}\psi\right) e^{-imc^2t/\hbar} \tag{38}$$

$$\frac{\partial^2\Phi}{\partial t^2} = \left(\frac{\partial^2\psi}{\partial t^2} - \frac{2imc^2}{\hbar}\frac{\partial\psi}{\partial t} - \frac{m^2c^4}{\hbar^2}\psi\right) e^{-imc^2t/\hbar} \tag{39}$$

Substituting into Eq. (36), the $m^2c^2/\hbar^2$ terms cancel:

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \frac{2im}{\hbar}\frac{\partial\psi}{\partial t} - \nabla^2\psi = 0 \tag{40}$$

In the non-relativistic limit where $|\partial^2\psi/\partial t^2| \ll (mc^2/\hbar)|\partial\psi/\partial t|$:

$$-\frac{2im}{\hbar}\frac{\partial\psi}{\partial t} - \nabla^2\psi \approx 0 \tag{41}$$

Rearranging:

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi} \tag{42}$$

This is the free-particle Schrödinger equation.

8.4 Adding Potentials

If the oscillatory frequency varies spatially, $\omega(x) = (mc^2 + V(x))/\hbar$, the same analysis yields:

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right)\psi \tag{43}$$

The Schrödinger equation describes envelope dynamics of an oscillatory field in the non-relativistic limit. For comparison, Nelson's stochastic mechanics (Nelson, 1966) derives the same equation from Brownian motion assumptions rather than from envelope dynamics. The Klein-Gordon-to-Schrödinger reduction via SVEA is well-established in nonlinear optics; Robson and Biancalana (Robson and Biancalana, 2021) derive Schrödinger-type envelope equations from relativistic field equations in an optics context, demonstrating that the mathematical step is standard. What is novel in our treatment is not the reduction itself but the interpretive claim: that the Schrödinger equation *is* the non-relativistic envelope dynamics of a physical oscillatory field, rather than merely sharing its mathematical form with optical envelope equations.

8.5 The Role of $\hbar$

In this framework, $\hbar$ is the conversion factor between:

- Frequency and energy: $E = \hbar\omega$

- Wavenumber and momentum: $p = \hbar k$

It sets the scale at which oscillatory structure becomes relevant to measurement. Systems with characteristic action $S \gg \hbar$ have oscillations too fast to produce quantum effects at typical resolutions; systems with $S \sim \hbar$ are in the quantum regime.

$\hbar$ is not derived—it is an empirical constant setting the frequency scale of matter oscillations.

9 Falsifiable Predictions

9.1 High-Intensity Corrections

The Born rule (Eq. 17) is a leading-order approximation. The full expansion is:

$$P_{\text{click}} = P_{\text{dark}} + \alpha|\Psi|^2 + \beta|\Psi|^4 + O(|\Psi|^6) \tag{44}$$

Proposition 9.1 (Testable Deviation)**.** At sufficiently high field intensities (where the weak-signal approximation breaks down), the $|\Psi|^4$ term becomes non-negligible. This predicts:

$$P_{\text{observed}} \neq P_{\text{Born}} \quad \text{for } |\Psi|^2 \gtrsim \sigma^2 \tag{45}$$

The coefficient β depends on detector characteristics and could in principle be measured by comparing detection rates at different intensities.

9.2 Resolution-Dependent Coherence

The sinc^2 factor (Eq. 27) predicts specific dependence of interference visibility on measurement timescale Δt:

$$\text{Visibility} \propto \text{sinc}^2\left(\frac{\omega_c \Delta t}{2}\right) \tag{46}$$

Interference should degrade predictably as measurement duration increases beyond the coherence time.

9.3 Experimental Considerations

Testing these predictions requires:

1. Ultra-high intensity sources (to probe the $|\Psi|^4$ regime)
2. Precise control of measurement timescales
3. Detectors with well-characterized threshold and noise properties

Current technology may not reach the required regimes, but the predictions are definite and falsifiable in principle.

9.4 Theoretical Constraints on Born Rule Modifications

The $O(|\Psi|^4)$ corrections predicted by our model require careful qualification in light of recent no-go results. Galley and Masanes (Galley and Masanes, 2017, 2018) have shown that *any* modification of the Born rule leads to violations of the purification and local tomography principles—structural features that are thought to be constitutive of quantum theory. Torres Alegre (Torres Alegre, 2025) strengthens this result by proving that nonlinear modifications to the Born rule enable superluminal signalling via quantum steering in any generalised probabilistic theory satisfying purification. Taken at face value, these results appear to rule out genuine $|\Psi|^4$ corrections to quantum probability.

We take this constraint seriously. The resolution is that our $O(|\Psi|^4)$ corrections are *not* modifications to the Born rule at the level of the quantum formalism. They are detector-response artefacts:

properties of the threshold detection process applied to the oscillatory field, not properties of the underlying probability calculus. The distinction is between (a) modifying the rule $P = |\langle\phi|\psi\rangle|^2$ that governs state-to-probability maps within quantum theory, which the Galley-Masanes and Torres Alegre results prohibit, and (b) deriving that the *empirical* detection rate of a specific apparatus deviates from $|\Psi|^2$ at high intensities because the detector's response function is nonlinear. Our model predicts (b), not (a).

In concrete terms: normalise the detection probability by integrating over all detection outcomes and the result recovers a proper probability measure consistent with the Born rule. The $|\Psi|^4$ terms arise because a particular detector's click rate is not a linear function of the true detection probability—they are analogous to detector dead-time corrections in photon counting experiments, not to modifications of quantum mechanics. This interpretation is consistent with Valentini's framework (Valentini, 1991), in which deviations from quantum equilibrium (the Born rule) are possible in principle but relaxation dynamics drive the system toward $|\psi|^2$; in our model, the detection-theoretic "deviations" persist only in the raw click statistics of a specific apparatus, not in properly normalised probabilities.

The falsifiable content of the prediction is therefore this: specific detectors operating at high intensities should exhibit systematic deviations from the Born rule *in their raw count rates*, and the form of those deviations should match the Taylor expansion derived in Section 4. This is a prediction about detection physics, not about quantum probability per se, and it does not conflict with the Galley-Masanes no-go results.

10 Numerical Simulation Designs

The analytical results of Sections 3–8 make specific, quantitative predictions that can be validated through numerical simulation before any experimental test is attempted. We present four simulation designs of increasing complexity, each targeting a distinct claim of the paper. Full implementation requires only standard numerical computing libraries (NumPy, SciPy).

10.1 Simulation I: Born Rule Emergence from Threshold Detection

10.1.1 Objective

Verify that threshold detection of a noisy complex signal produces detection probabilities proportional to $|\Psi|^2$ at leading order, with quantifiable $O(|\Psi|^4)$ corrections at high intensity.

10.1.2 Physical Setup

A complex signal $\mu = \frac{1}{2}\Psi$ is embedded in circular complex Gaussian noise $\eta \sim \mathscr{CN}(0,\sigma^2)$. A detector fires when $|A|^2 = |\mu+\eta|^2 > \Theta$.

10.1.3 Algorithm

1. Fix noise variance $\sigma^2 = 1.0$ and threshold Θ.
2. For each amplitude $|\Psi| \in \{0, 0.01, 0.02, \ldots, 3.0\}$:
   (a) Generate $N = 10^6$ independent noise samples $\eta_k \sim \mathscr{CN}(0,\sigma^2)$
   (b) Set $\mu = \frac{1}{2}|\Psi|$ (real-valued without loss of generality)
   (c) Compute $|A_k|^2 = |\mu+\eta_k|^2$ for each sample
   (d) Record $\hat{P}_{\text{click}}(|\Psi|) = \frac{1}{N}\sum_k \mathbf{1}[|A_k|^2 > \Theta]$
3. Fit the resulting curve to:
$$\hat{P}_{\text{click}} = \hat{P}_{\text{dark}} + \hat{\alpha}|\Psi|^2 + \hat{\beta}|\Psi|^4 \tag{47}$$
   using nonlinear least squares.
4. Repeat for multiple threshold values $\Theta \in \{0.5, 1.0, 2.0, 3.0\}$.

10.1.4 Parameters

Parameter	Value	Justification
σ^2	1.0	Normalised noise floor
Θ	0.5, 1.0, 2.0, 3.0	Spans low to high threshold regimes
$\lvert\Psi\rvert$ range	$[0, 3.0]$	Covers weak-signal ($\lvert\Psi\rvert \ll \sigma$) to strong
N per point	10^6	$\sim 0.1\%$ statistical uncertainty
Amplitude steps	301	Fine resolution for polynomial fit

10.1.5 Expected Outcomes

- For $|\Psi| \ll \sigma$: $P_{\text{click}} \approx P_{\text{dark}} + \alpha|\Psi|^2$ (Born rule regime). The coefficient α varies with Θ but the quadratic form is universal.
- For $|\Psi| \sim \sigma$: Measurable deviation from pure quadratic; the $\beta|\Psi|^4$ term becomes significant. This is the falsifiable prediction.
- P_{dark} should equal the Marcum Q-function $Q_1(0, \sqrt{\Theta/(\sigma^2/2)})$, providing an analytical cross-check.
- The ratio β/α should be negative (saturation effect), calculable from the noncentral chi-squared distribution.

10.1.6 Validation Criteria

The simulation succeeds if: (i) $R^2 > 0.999$ for the quadratic fit in the weak-signal regime ($|\Psi| < 0.5\sigma$); (ii) residuals from a pure-quadratic fit show systematic positive-then-negative structure, confirming $O(|\Psi|^4)$ corrections; (iii) fitted P_{dark} matches the Marcum Q-function prediction to within Monte Carlo error.

10.2 Simulation II: Interference Pattern Recovery

10.2.1 Objective

Demonstrate that two-path superposition of oscillatory fields, processed through demodulation and threshold detection, produces the standard double-slit interference pattern.

10.2.2 Algorithm

1. Define a one-dimensional detector screen at positions $x \in [-L, L]$.
2. For each screen position, compute the two-path envelope:

$$\Psi(x) = \psi_0 \left(e^{ikd \sin\theta_1(x)} + e^{ikd \sin\theta_2(x)} \right) \tag{48}$$

   where $\theta_{1,2}(x) = \arctan((x \mp d/2)/D)$, d is slit separation, D is slit-to-screen distance, and $k = 2\pi/\lambda$.
3. At each position, generate $N = 10^5$ noise samples $\eta_k \sim \mathscr{CN}(0, \sigma^2)$.
4. Compute detection counts: $C(x) = \sum_k \mathbf{1}[|\frac{1}{2}\Psi(x) + \eta_k|^2 > \Theta]$.
5. Compare $C(x)/N$ against:
   - Born rule prediction: $P \propto |\Psi(x)|^2 = 4|\psi_0|^2 \cos^2(\delta(x)/2)$
   - Full threshold detection prediction (including $O(|\Psi|^4)$)
6. Run a "which-path" variant: at each position, randomly assign $\Psi(x) = \psi_1(x)$ or $\Psi(x) = \psi_2(x)$ with equal probability. Verify interference destruction.

10.2.3 Parameters

Parameter	Value	Justification
λ	1.0 (normalised)	Characteristic wavelength
d (slit separation)	5λ	Multiple fringes visible
D (screen distance)	100λ	Far-field regime
L (screen half-width)	50λ	Covers $\sim$10 fringes
$\lvert\psi_0\rvert$	0.2	Weak-signal regime
σ^2	1.0	Standard noise
Θ	1.0	Moderate threshold
Screen points	500	Sufficient spatial resolution
N per point	10^5	Statistical adequacy

10.2.4 Expected Outcomes

- Coherent superposition: sinusoidal fringe pattern matching $\cos^2(\delta/2)$ prediction.
- Which-path (mixture): uniform sum $|\psi_1|^2 + |\psi_2|^2$ with no fringes.
- Fringe visibility $V = (C_{\max} - C_{\min})/(C_{\max} + C_{\min})$ should approach 1.0 for equal-amplitude paths and decrease predictably with amplitude asymmetry.

10.3 Simulation III: Resolution-Dependent Coherence

10.3.1 Objective

Verify the $\mathrm{sinc}^2(\omega_c \Delta t/2)$ dependence of measured intensity on demodulation window duration, and demonstrate the continuous quantum-to-classical transition.

10.3.2 Algorithm

1. Generate a carrier signal $\Phi(t) = \mathrm{Re}[\Psi_0 \cdot e^{-i\omega_c t}]$ over a time interval $[0, T]$, sampled at rate $f_s \gg \omega_c/(2\pi)$.
2. For each window duration Δt in a logarithmic sweep from $0.01/\omega_c$ to $100/\omega_c$:
   (a) Apply rectangular window $g(t)$ of width Δt
   (b) Compute demodulated amplitude: $A = \sum_n g(t_n - t_0) \cdot \Phi(t_n) \cdot e^{+i\omega_c t_n} \cdot \delta t$
   (c) Record $|A|^2$ (averaged over multiple window placements t_0)
3. Normalise by the SVEA prediction ($|\Psi_0|^2/4$).
4. Overlay the analytical prediction $\mathrm{sinc}^2(\omega_c \Delta t/2)$.
5. For the coherence test: repeat with a two-frequency superposition:

$$\Phi(t) = \mathrm{Re}[c_1 \phi_1 e^{-i\omega_1 t} + c_2 \phi_2 e^{-i\omega_2 t}] \tag{49}$$

   Demodulate at $\omega_c = \omega_1$ and observe how the cross-term $|c_1 \phi_1 + c_2 \phi_2 e^{-i\Delta\omega \cdot t_0}|^2$ averages away as Δt increases.

10.3.3 Parameters

Parameter	Value	Justification
ω_c	$2\pi \times 10^3$ rad/s	Arbitrary carrier frequency
f_s	$100 \times \omega_c/(2\pi)$	Well above Nyquist
$\lvert\Psi_0\rvert$	1.0	Normalised
Δt range	$[10^{-2}, 10^2]/\omega_c$	Four decades spanning all regimes
Δt points	200 (log-spaced)	Smooth curve
t_0 averages	100	Reduce placement variance
$\Delta\omega/\omega_c$	0.01, 0.1, 0.5	Different decoherence rates

10.3.4 Expected Outcomes

- Single-frequency: $|A|^2/|A_{\text{SVEA}}|^2$ follows $\text{sinc}^2(\omega_c \Delta t/2)$ exactly.
- $\Delta t \ll \pi/\omega_c$: ratio approaches 1 (high resolution, classical-like).
- $\Delta t \gg \pi/\omega_c$: ratio decays as $(\omega_c \Delta t)^{-2}$ (low resolution).
- Two-frequency case: interference visibility decays as $\text{sinc}(\Delta\omega \cdot \Delta t/2)$, demonstrating measurement-induced decoherence without environmental interaction.

10.4 Simulation IV: Full Pipeline—Oscillatory Field to Detection Statistics

10.4.1 Objective

Integrate all components (field generation, demodulation, noise injection, threshold detection) into a single Monte Carlo pipeline that reproduces standard quantum measurement predictions from purely classical signal-processing operations.

10.4.2 Algorithm

1. **Field generation**: Create a 1D spatial field $\Phi(x,t) = \text{Re}[\Psi(x)e^{-i\omega_c t}]$ where $\Psi(x)$ is a Gaussian wave packet:
$$\Psi(x) = \left(\frac{1}{2\pi\sigma_x^2}\right)^{1/4} \exp\left(-\frac{(x-x_0)^2}{4\sigma_x^2} + ik_0 x\right) \tag{50}$$
2. **Temporal sampling**: At each spatial position x_j, sample $\Phi(x_j,t)$ at rate f_s over duration T.
3. **Demodulation**: Apply matched filter (rectangular window, carrier multiplication) to extract $A(x_j)$.
4. **Noise injection**: Add $\eta \sim \mathscr{CN}(0,\sigma^2)$ to each $A(x_j)$.
5. **Detection**: Record click if $|A(x_j)+\eta|^2 > \Theta$.
6. **Statistics**: Repeat N times per position. Build histogram of detection counts vs. position.
7. **Comparison**: Overlay normalised detection histogram with $|\Psi(x)|^2$ (Born rule prediction).

10.4.3 Test Cases

1. **Single Gaussian packet**: Detection histogram should match $|\Psi(x)|^2 \propto \exp(-(x-x_0)^2/(2\sigma_x^2))$.
2. **Superposition of two packets**: $\Psi = \Psi_1 + \Psi_2$ with spatial separation. Interference fringes should appear in the overlap region.
3. **Energy eigenstate superposition**: $\Psi(x,t) = c_1\phi_1(x)e^{-i\omega_1 t} + c_2\phi_2(x)e^{-i\omega_2 t}$. Time-averaged detection should yield $|c_1|^2|\phi_1|^2 + |c_2|^2|\phi_2|^2$ (no interference), reproducing decoherence.

10.4.4 Parameters

Parameter	**Value**	**Justification**
ω_c	$2\pi \times 10^4$ rad/s	High carrier
σ_x	10λ	Localised packet
k_0	$2\pi/\lambda$	Central wavenumber
Spatial range	$[-50\lambda, 50\lambda]$	Covers packet $\pm 5\sigma_x$
Spatial points	500	
σ^2 (noise)	1.0	Normalised
Θ	1.0	Moderate threshold
N (trials)	10^5	Adequate statistics
f_s	$50\omega_c/(2\pi)$	Oversampled
Δt (window)	$10 \times 2\pi/\omega_c$	SVEA regime

10.4.5 Expected Outcomes

- Detection histograms match $|\Psi(x)|^2$ to within statistical error for weak signals.
- Systematic deviations at high $|\Psi|$ (packet peak) consistent with $O(|\Psi|^4)$ corrections.
- Superposition fringes appear and disappear depending on whether coherence is maintained (same frequency) or broken (different frequencies + time averaging).
- The entire pipeline uses only: FFT, random number generation, and threshold comparison—no quantum postulates are invoked at any stage.

10.5 Computational Requirements

All four simulations are computationally tractable on a modern workstation. Simulation I requires approximately 3×10^8 random samples (a few seconds). Simulation II requires 5×10^7 samples across 500 spatial points. Simulation III involves signal processing rather than Monte Carlo, with the main cost being the FFT operations ($\sim$1 minute). Simulation IV is the most demanding at approximately 5×10^7 total trials, but remains well within single-machine capability ($\sim$10 minutes with vectorised NumPy operations).

Implementation code and reproducibility scripts will be made available upon publication.

11 Discussion

11.1 What This Framework Explains

1. **Born rule**: Derived from threshold detection statistics
2. **Interference**: Automatic from phase correlations
3. **Uncertainty**: Nyquist/Gabor aliasing limit
4. **Schrödinger dynamics**: Non-relativistic envelope of oscillatory field
5. **Decoherence**: Frequency differences eliminate cross-terms
6. **Measurement "collapse"**: Transition from low to high resolution sampling

11.2 What This Framework Does NOT Explain

1. **Bell inequality violations** (Bell, 1964): Multi-particle entanglement correlations exceed classical limits. Our single-system detection model does not address this.
2. **Spin**: Intrinsic angular momentum requires additional structure (possibly phase winding or internal degrees of freedom).

3. **Identical particles**: Fermi-Dirac vs. Bose-Einstein statistics are not derived.
4. **Ontology of Φ**: We do not commit to what the oscillatory field "is." It could be fundamental, or an effective description of deeper structure.

11.3 Relation to Bell Inequalities

The Bell inequality violations observed in quantum mechanics (Bell, 1964), confirmed experimentally with increasing rigour from Aspect, Dalibard, and Roger (Aspect et al., 1982) to recent loophole-free tests, require correlations that exceed what any local hidden variable theory can produce. Our framework, as presented, is a single-system detection model. Extending it to multi-particle entanglement would require:

- Configuration-space dynamics (Bohm, 1952)
- Retrocausal boundary conditions (Cramer, 1986)
- Contextual hidden variables (Khrennikov, 2009b)
- Threshold detection of correlated classical fields (Khrennikov, 2012a)

Khrennikov's demonstration that PCSFT can reproduce CHSH violations through properly calibrated threshold detection of classical random signals is particularly relevant, as it suggests that our detection-theoretic framework may extend to multi-particle correlations more naturally than standard hidden variable models. We make no claims about which (if any) of these extensions is correct, but the threshold detection route appears the most natural continuation of the programme developed here.

11.4 Ontological Interpretation

The oscillatory field model is compatible with multiple ontological stances:

- **Realist**: Φ is the fundamental physical reality; particles are patterns
- **Instrumentalist**: Φ is a useful calculation device; ontology is irrelevant
- **Structural**: The oscillatory structure is what matters, not its "substance"

This paper is neutral between these interpretations. The derivations hold regardless of one's preferred metaphysics.

12 Conclusion

We have derived the Born rule $P \propto |\psi|^2$ from three ingredients: an oscillatory field substrate, finite-resolution demodulation, and threshold detection with noise. The derivation uses only standard signal processing and detection theory.

The key results are:

1. The Born rule is the leading-order term in a Taylor expansion of detection probability
2. Higher-order corrections ($|\psi|^4$) provide falsifiable predictions
3. Quantum interference emerges automatically from phase correlations
4. Heisenberg uncertainty IS the Gabor limit—aliasing, not metaphysics
5. The Schrödinger equation describes envelope dynamics of an oscillatory field

This does not solve all interpretational problems. Bell inequality violations and multi-particle entanglement require additional theoretical structure not provided here. But for single-system quantum mechanics, the Born rule is not mysterious: it is what you get when you sample oscillatory structure through a noisy threshold detector.

Probability is not fundamental. It is what oscillation looks like from inside.

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